**Working title:**

**Causal effect under spatial interference with**

**numerical simulation and case studies**

1. **Background**

In the context of causal inference, the concept of causal ‘interference’ refers to the existence of a dependency of a given unit’s outcome on the treatments of other units. Interference is common in observational studies, and also in social science RCTs. Since it describes a statistical dependency, it can take many forms and arise from multiple mechanisms. In Rubin’s (1978) original articulation of the Potential Outcome (PO) framework, no interference is one of the basic identification assumptions. Later on, no interference is widely known as one component of the stable unit treatment assumption (SUTVA), unadjusted violation of which leads to biased causal effect estimations. Nowadays, methods to adjust for interference in causal inference continues to be developed in the statistics community.

Spatial interference is, broadly speaking, scenarios of causal interference involving spatial interaction among study subjects. Spatial interaction is a major source of interference. Well known examples include: air pollution treatments where pollution travels to non-treated areas; neighbourhood policing and crime where policing in one area may affect crimes in others; vaccination where an increased proportion of vaccination rate could protect the unvaccinated, and survey experiments on social networks where information may propagate on the network. In these cases, SUTVA cannot be sufficiently or efficiently guaranteed through randomisation design. And therefore, statistical adjustment methods are needed.

There are trivial and non-trivial cases of spatial interference. For example, consider five typic scenarios. Scenario 1: interference arises due to spatial confounding. Scenario 2: interference arise due to topological ‘confounding’. Scenario 3: Mediation by a spatial factor at the same scale or a finer scale compared to the scale of analysis. Scenario 4: A global mediator exists. Scenario 5: Spillover of treatment among units. Under these scenarios, the statistical dependence manifested may present as spatial interference. Scenario 1 and 2 may be considered trivial, as adjusting for confounding will be able to alleviate the interference. The other scenarios are non-trivial, in the sense that, the interference may point to factors of substantial significance to the causal relation under study. Of these three scenarios, here we focus on the spillover of treatments. This is the most common type of spatial interference.

1. **Research objectives**

The basic setting of the proposed study is causal inference with spatial interference based on observational data. There are two main objectives:

(1) Clarify some connections between causal interference and reduced form spatial econometrics. E.g. What are the common grounds and discrepancies in the formalism of direct/ indirect effect, or the assumptions and interpretations?

(2) Explore promising connections to spatial methods. Extend nonparametric techniques for retrieving causal effects and bias corrected confidence intervals, robust to spatial distribution shift. Demonstration on numerical simulation and house prices.

1. **Literature**

**General interference**

In the general PO literature, causal effects under interference are often defined in this set up: For individuals , a binary treatment is randomly assigned. The treatment status of neighbours is referred to as the treatment program . The presence of interference means that the potential outcome for unit depends not only on its own treatment but also . If each combination of neighbour treatment status is considered a different treatment level, then there could be as many as potential outcomes for unit (Rubin 1978, 1990). To keep the dimension of the problem tractable, it’s often assumed . That is, the potential outcome under treatment level and are equal conditional on treatment and a function of its neighbor treatment program . This is also known as an adapted version of SUTVA (citation). Under this condition, the individual level direct treatment effect can be expressed as: holding neighbour treatment program constant. can be obtained by marginalizing out *g* (Sobel, 2006). The indirect effect can be expressed as: (Halloran, 1995) or (Wagers).

The estimation that follows from this definition takes several directions. In general, some knowledge is required to match true treatment levels to the outcomes. First, true exposure levels can be empirically measured via exposure mapping (e.g. Aronow & Samii, 2017). Depending on the cases, this is not always possible. An alternative is to rely on assumptions about the structure of the studied causal relation and employ generalized propensity score based methods. The generalized propensity is defined as ) the probability of treatment given self and neighbour characteristics (Forastiere et.al. 2020). Inverse probability weighting (IPW) techniques can then be used to retrieve direct causal effect with correctly estimated propensity scores. But still, the estimation of indirect effects is most challenging and lacks well tested frameworks.

**Spatial interference**

Under the general PO framework, spatial interference can be accommodated in the same principle as other general forms of interference. In practice, the generalized propensity score route is more commonly taken. A few aspects mark spatial interference as distinct from other forms: there could be spatial confounding that needs to be addresses at the same time; physical distance could be incorporated to the advantage of the analysis (e.g. Giffin et.al., 2020).

At the same time, there are more specialised methods to handle spatial interference. For example, in the case of evaluating air pollution interventions, simulation of air pollutants diffusion can substitute for field-based exposure mapping (Zigler et.al. 2021). In the case of evaluating the effect of subsidies, economic theory based structural models can be used to construct mediating factors to feed into the causal effect estimation (Wager et.al. 2021). These approaches enrich the standard PO framework by leveraging domain knowledge about the subject of study to construct theoretical measurements.

**Limitations in existing literature**

It is difficult to extend the PO framework to accommodate general interference, both in terms of definition and techniques. Discussion missing: Does adding more structure to PO make it some reinvention of reduced form (spatial) econometrics? It’s a good time to think about this connection.

Knowledge is lacking besides measures of direct effects. How to decide which neighbors to include? Is propensity scores accurate when we do not know exactly the structure of causal interaction? Is there an endogeneity problem in causal interaction patterns? How does a shift in dependence structure affect causal effect overall and locally? Would we be able to generalize to out-of-sample counterfactuals?

On spatial interference more specifically, existing methods make proximation of unit-wise treatments by introducing structures of spatial causal interaction, often expressed as a non-restrictive spatial network or spatial random field. There is space for exploration, in term of how to make estimation more efficient and realistic by trying to describe the structure of causal interaction rather than using null structures.

**Proposed extension to literature and challenges**

It may be informative if we can learn the interaction structure; or learn the treatment levels directly. Sparse and more realistic representation of exposure means: more efficient estimation of causal effects；increased robustness.

There will be several challenges in pursuing this direction: will the inferred causal interaction structure be endogenous? Will the learning be expensive? Is there theoretical guarantee for the approach? All these questions need to be addressed.

Accordingly, the proposed study will explore possible solutions by borrowing strength form new techniques in the field of causal inference more broadly. This includes applying search algorithms or network reconstruction techniques for learning causal interaction graph structure (e.g. Bhattacharya, 2020); using flexible non-parametric techniques to estimate causal effects (e.g. Athey, 2018); and using conformal inference based sensitivity analysis to retrieve correct confidence intervals for the non-parametric estimators (e.g. Jin, Ren & Candes, 2021).

To summarize, the project aims to clarify some connections between ‘causal inference under interference’ and reduced form spatial econometrics. It intends to use flexible prediction models to estimate the causal effect under interference and use sensitivity models to assess the uncertainty of these counterfactual predictions. The proposed approaches will be tested on simulation studies and real datasets.

**Proposed work in relation to prior work on conformal inference with nonexchangeability and for causal inference**

The main idea in the referenced papers is to introduce weighting functions into the shuffling of observations. This project departs from these existing efforts and tries to fold a spatial propensity score weighting into the conformal procedure's weighting function. This is a simple and straight forward way of 'spatializing' the conformal causal inference procedure. The theoretical properties of the design can be understood in terms of conformal inference with non-exchangeable data points (see recent work of Candes et.al. for theoretical justification). This line of inquiry has been taken up in times series data based conformal inference, it has yet to be applied to spatial data, which implies a different structure of dependence. The spatial dependence structure differs from time series data in several important ways:

(1) The dimension of dependence structure:

In a time series, observations are dependent on previous observations. This an auto-correlation structure with only one dimension. In contrast, the spatial dependence structure (e.g. in our case the spillover of treatments) is two dimensional. First, this adds to the complexity of weighting functions in the estimation procedure. Also, there is potentially a feedback issue. IN time series, all data points affect the data points that are observed later in time. There is no influence from future data points to past data points. In the spatial case, this unidirectional influence structure does not hold. It is likely that a location affects its nearest neighbours and vice versa, which could lead to a feedback structure. This is well-understood and studied in the spatial econometrics literature. The common assumption is that the feedback settles at some level of equilibrium. In the causal inference case however, existing methods to handle spillover effect typically ignore the potential feedback. In other words, it is assumed that the treatment in one location may affect the outcome of another, possibly through a spillover treatment. But this spilled-over treatment is not going to affect the ego location in retrospect. This is plausibly realistic in binary treatments, where the outcome may not be overly sensitive to the dosage of treatment, ans therefore the treatment level could be discretised as treated/ spill-overed/ untreated. In this study, we follow this tradition as we are working with binary treatments in our empirical cases. Still, it should be noted that, the underlying assumptions may need to be revisited in other scenarios.

(2) The stationarity of dependence structure:

The weighted conformal inference technique works best when the auto-correlation structure is stationary over time. This is similar to the spatial dependence case. The stationarity of process serves to guarantee that the 'local' exchangeability of data points in blocks are valid and equally so through all the samples. If stationarity is violated, it could mean that under the same predefined dependence structure, some data points are not actually exchanegable and therefore shuffling them leads to biased prediction intervals. In the time series case, stationarity is relatively easier to achieve and common in empirical studies such as in the fields of economics. In the spatial case, however, the stationarity assumption is very strong and is more likely to be violated in empirical observational data.

(3) Obtaining and representing the dependence structure:

This block structure in data with dependence structures can be obtained prior to the inference either from data or based on substantial knowledge of the data generating and sampling scheme. In the time series case, the block structure can be (relatively) reasonably defined by the time periods. In the spatial case, this can be achieved in several ways. For example, spatial block units can be defined as spatial clusters obtained from exploratory analysis or predefined spatial units. Or in our case, spatial dependence structure can be implied by the spatial propensity of treatment, which provides a continuous measure of exchangeability across spatial data points.

(4) Endogeneity of dependence structure:

In predictive analysis, as is often the case with conformal procedures in existing methodological and empirical literature, endogeneity of dependence structure is not a serious concern. However, in the case of causal inference, more consideration is required in this regard. With time series data, the relatively simple auto-correlation structure imposes little restrictions/ encodes limited amount of information. This is preferable in causal inference, as it means lowers chanes of bias due to over fitting the data. In spatially dependent data, on the other hand, if the dependence structure is obtained from observations, there could be substantial endogeneity due to the spatial structure being related to/ a consequence of the causal process under investigation. Respectively, encoding the dependence structure would be, in a sense, leaking the response variable and therefore creating bias in the estimation/ prediction. To the best of my knowledge, there is no perfect solution to this issue when working with observational data. And this could even affect randomized experiments. In existing literature, the spatial endogeneity issue is typically handled by using non-restrictive spatial structures. This could be distance-based kernel functions or spatial networks. The idea is that, since geographic distance is an exogenous factor, using the decay in spatial phenomenon to account for dependence is less problematic. This is similar to the time periods based auto-correlation structure that is often found in time series based studies. Alternatively, if using data generated spatial structure, the bias introduced is hard to verify or quantify. In this study, we will explore both distance decay based spatial structure and data driven definitions of spatial dependence. The former is characterized by a distance based kernel in the weighting functions; the latter, spatial propensity score based weighting functions.

Overall, the general principles of time series conformal procedure still applies, but it should only work well under some strong assumptions about the spatial dependence structure.

**Case study: simulation**

The basic setup for the simulation study follows the tradition of earlier studies. There are *N* observational units =1, …, n generated on a 30x30 grid. Each unit is characterised by covariates X. is mean zero, variance 1 Gaussian process. The treatment Z can be binary or continuous. It is randomly assigned to units, independently determined by . The unit level outcome Y is determined by both unit level treatment and neighbour treatments, resembling a case of spatial interference due to spillovers. The spillover is specified in two ways. First, it is specified as a function of physical distance between units, realised as kernel smoothing . Second, it is specified as spillover through a network structure between units, so that . is jointly determined by and , with independent standard normal noise . From the basic setup, confounders *H* and unknown confounders *U* can also be introduced to resemble realistic scenarios in observational studies. The proposed procedures will be tested on the generated datasets. The performance will be evaluated under test datasets generated with different spatial parameters.

**Case study: Chicago crime rates**

Intro

To provide benchmark on real data, the Chicago community policing and crime rates case will be used. As crime rates skyrocketed in the 1980s, community policing became increasingly popular in the United States. On April 29, 1993, the city of Chicago introduced community policing in what became known as the Chicago Alternative Policing Strategy (CAPS). The department initiated the program in 5 of the city’s 25 police districts (jointly selected by the police department and the mayor’s office: Englewood, Marquette, Austin, Morgan Park and Rogers Park). By the end of 1995, CAPS was operating in every police district in Chicago. This creates an opportunity to evaluate the spillover effect of neighborhood policing on precincts that have not adopted the intervention. The case has been previously studied in several papers (e.g. Verbitsky-Savitz & Raudenbush, 2012). Crime rates affected by the CAPS can be evaluated on existing annual data.

Data and design

This case study uses existing public datasets. Main dataset is 1990s historical records of annual beat and precinct level reported personal crimes in Chicago (citation). Population density to normalize crime counts into crime rates will be taken from US Census Bureau datasets (citation) and interpolated to necessary spatial granularity.

**Case study: UK house prices**

Intro

A second case study is to measure the effect of location specific liquidity shocks on house prices in the UK. The price of residential properties is an important indicator for any economic system. In the UK, like in many other economies, property prices are unevenly distributed geographically. For decades, this phenomenon has been the focus of theoretical and empirical investigations. In observational studies, even with panel based econometric models, the changes in house prices are usually endogenous to the UK economy and therefore it is difficult to establish causal interpretations. To overcome this issue, we choose to study the effect of plausibly exogenous shocks on the UK housing market. This shall add to our existing understanding of the dynamics of the UK housing market. Further, the proposed methods in this study will be used to appropriately handle the spatial nature of the phenomenon, providing robust and interpretable causal inference.

More specfically, international oil price will be used as the source of exogenous shock in this study. When oil price rises, this affects locations differently. For the few oil industry hubs in the UK, inflated oil price means increase in real and expected income, which then translates to inflated house prices. For other locations, this effect does not exist. But places close to oil industry hubs may receive a spillover effect in the house price inflation. The proposed methods will be used to measure both the direct and indirect effect of oil price shock on UK house prices. The evidence adds to our existing understanding of the financialization of housing market and co-movements of prices within UK.

Brief review of literature on housing price dynamics:

There are three main strands of relevant empirical research.

(1) the rippling effect of housing prices

The first tests for convergence across regional house prices (e.g. Holmes & Grimes, 2008; Montagnoli & Nagayasu, 2015). The second studies the ripple effect of prices (e.g. Hudson et.al., 2018). These two themes are united under neoclassical economic theories.

The rippling effect of regional housing prices has long been studied in the fields of economics and regional science. According to the new classical economics theory tradition, local shocks on housing prices spreads out across space and eventually converge to a state of equilibrium (citation). This point of view has been studied empirically in the UK with mixed results. Although the diffusion of housing prices changes seem to exist (citation), and with a certain structural relationship of leading and following (citation), it is less conclusive as to whether the regional house prices are converging (citation). Meanwhile it is still meaningful to study the dynamics if housing price movements, especially with regard to understanding the increasing financialisation of housing market and its potential impact on the stability of the economy (citation).

(2) Financialisation of housing and the role of international influence on the UK housing market:

The third strand (e.g. Hamnett & Reades, 2019; McMillan, 2011; Schindler, 2014), which this essay is most closely related to, looks outside the equilibrium assumptions, and study short-mid-term house market dynamics that are informative for the policy makers and average investors. There are several limitations in the empirical literature: Most studies are based on regional data. Many studies focus on long-run equilibriums and ignore short-term dynamics. We have limited knowledge on how exogenous sources of fluctuation impacts the housing market. Respectively, this study intends to add to the literature by offering local authority level analysis under exogenous shocks, which could shed new light on the behaviour of UK house market. (...More on the role of money supply and mortgage finance in driving up housing price in the short run...)

(3) previous studies on the oil price shocks to housing markets, causal effect and mechanisms.

The oil price shock on house prices has been previously studied in countries including Canada and the US. From these studies, we can have a basic picture of the causal mechanisms delivering the effect at question. The effect works through two main channels. First, when the oil price rises, this translates to a rise in real income in the oil industry (citation). Second, the rising oil price signals a better credit to get financial resources such as mortgage to secure house purchases (citation). Although this effect as not been systematically studied in the UK before, some recent publications suggested that similar mechanisms may be at work in some UK areas as well (citation).

Datasets and design

Main datasets for the empirical case here are house price measures, oil price measures and other control variables. For house prices with sufficient temporal and geographic granularity, the UK land registry data (data.gov.UK, 2022a) or the CDRC MIAC house price data will be used (CDRC, 2022). The former contains local authority level monthly HPI. The latter contains county level monthly HPI. For oil prices, the US Energy Information Administration (2022) historical records for Brent oil price (dollar per barrel) will be used. The UK oil and gas industry is concentrated in several locations, including Aberdeen and southeast England. Exact locations are defined based on the employment size in the oil and gas industry as a share of the local labour market. This information is taken from public datasets (data.gov.UK, 2022b). Following existing literature, two other variables are included to control for the supply and demand factors driving local prices. The demand factor is represented by the yearly growth of population (data.gov.UK, 2022c). The supply factor is represented by the yearly dwelling supply index (data.gov.UK, 2022d).

preliminary results

The estimated effect in this study is: when the international oil price increases, this translates to growth in real and expected income in UK oil industry hubs, which translates to a liquidity shock to the local house markets and inflated prices (Lorusso & Pieroni, 2018). This effect could then spillover to the HPI of neighbouring areas. The results are mostly in line with the proposed hypothesis. For the periods studied, the oil hub indicator is positively associated with HPI growth. This means that a positive oil price shock influences UK oil industry hubs’ HPI movement. Also, the distance variable is negatively associated with HPI growth. This means that, the further away a location is from UK’s oil industry hubs, the weaker its HPI growth is associated with international oil price fluctuations.

interpretations

The dynamics of property prices are an important consideration for private and public investors, as well as monetary policy makers and financial regulators. In the cases of liquidity shocks studied, we find significant geographic variation in the local house market responses. The propagation of such shocks through the local submarkets indicates important structural characteristics of the UK house market. The evidence helps us to build a better understanding of the determinants and dynamics of UK house price movements.

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